Spiral breakup due to mechanical deformation in excitable media

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To address the problem of how cardiac muscle contraction affects the dynamics of rotating spiral waves, spiral breakup induced by mechanical deformation in excitable media is studied in two partial-differential-equation models. It is shown that spirals begin to break up at $\omega=0.5\omega_0$ when we increase the amplitude of the mechanical deformation gradually. Our numerical results point to a new mechanism of transition from spirals to spatiotemporal chaos, in which the anisotropic time-dependent diffusion coefficient is essential.

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I. INTRODUCTION

Excitable media represent a wide class of nonequilibrium systems that play an important role in physical, chemical, and biological applications. Spiral excitation waves in two spatial dimensions are among the most paradigmatic examples of spatiotemporal self-organizing structures in excitable media. They have been the subject of extensive research in a large variety of systems, which include the catalytic surface processes, the Belousov-Zhabotinsky (BZ) reaction, and the heart muscle [1–3]. The breakup of spiral waves in excitable medium disorders the spatial pattern of excitation and results in turbulent or chaotic behavior. Understanding of transitions from spiral waves to defect-mediated turbulence has been of great interests in nonlinear physics. From a practical point of view, understanding the mechanism of spiral instability could have potential impacts in cardiology. Recent studies on animal hearts show clear evidences that the transitions from a state of ordered spiral waves to a state of defect-mediated turbulence are responsible for life-threatening situations such as tachycardia and fibrillation [4–10]. This transition (spiral breakup) has been found in experiments in pattern forming chemical reactions [11–13] and numerical simulations in various models [14–20]. Recently, the transition from spiral waves to defect-mediated turbulence induced by gradient effects in a reaction-diffusion system was also studied in experiment and numerical simulation [21]. On the other hand, spiral will also break up under external perturbations: Belmonte *et al.* [22] observed a transition from spiral to spatiotemporal chaos when a periodic external forcing is added and the ratio of the spiral-rotation period to that of the forcing is close to 3/2; Biktashev *et al.* [23] studied the influence on spiral of linear shear flow and found that it can induces spiral breakup.

To investigate how cardiac muscle contraction affects the dynamics of rotating spiral waves, Muñuzuri *et al.* [24] designed an elastic excitable medium by incorporating the BZ reaction into a polyacrylamide-silica gel to investigate the effect of mechanical deformation on spiral waves. They reported that for *equal* frequencies of deformation and spiral rotation, spirals will drift. Recently, we derived, directly from the original reaction-diffusion equation and its spiral wave solution, an approximate formula of the perturbation-induced spiral wave drift velocity. And we are able to explain the main features appearing in the spiral constant drift induced by periodic mechanical deformation [25]. However, it is more important to study whether spiral will break up under mechanical deformation because cardiac muscle is contracting all the time and the transitions from spiral waves to defect-mediated turbulence are responsible for life-threatening situations.

In this paper, we will study the effects of mechanical deformation on spiral waves in excitable media. It will be shown that when the amplitude of mechanical deformation is above a critical value and the frequency of mechanical deformation is around 0.5 times of that of the spiral, the transition from spiral waves to defect-mediated turbulence induced by mechanical deformation will happen. The mechanism of this kind of spiral breakup is intuitively understood: the wave is broken when the front curvature exceeds a certain critical curvature [26,27] which is modulated by periodic mechanical deformation.

II. SPIRAL BREAKUP IN THE FITZHUGH-NAGUMO MODEL

A. The FitzHugh-Nagumo model with mechanical deformation

Let us begin with a model of two-dimensional cardiac tissue with transmembrane current described using simplified excitable dynamics of the FitzHugh-Nagumo-type [28]

$$\frac{\partial e}{\partial t} = \nabla^2 e - ke(e-a)(e-1) - eg,$$

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$$\frac{\partial g}{\partial t} = \varepsilon(e,g)(-g - ke(e-a-1)), \tag{1}$$

where $\varepsilon(e,g)=\epsilon+\mu_1 g/(e+\mu_2)$. Here the variable e stands for the transmembrane potential, and variable g stands for the conductance of the slow inward current. The function $-ke(e-a)(e-1)$ in the first part of Eq. (1) determines the fast processes, such as the initiation of the upstroke of the action potential. The dynamics of the recovery phase of the action potential are determined by the time course of the variable g, mainly by the function $\varepsilon(e,g)$. The particular parameters in this model do not have a clear physiological meaning but are adjusted to reproduce key characteristics of cardiac tissue, such as shape of action potential, refractoriness, and restitution of action potential duration.

The mechanical deformation of medium can be modeled by an operation where any fixed point x of the medium is changed to $\overline{x(t)}$. Here, we consider a simple oscillation [24] $\overline{x(t)}=x[1+A\cos(\omega t+\phi)]$ and Eq. (1) is then modified to

$$\frac{\partial e}{\partial t} = e_{\overline{xx}} + e_{yy} - ke(e-a)(e-1) - eg,$$

$$\frac{\partial g}{\partial t} = \varepsilon(e,g)(-g - ke(e-a-1)). \tag{2}$$

Using the relations

$$e_{\overline{xx}} = \frac{\partial}{\partial \bar{x}}\left(\frac{\partial e}{\partial \bar{x}}\right) = \frac{\partial x}{\partial \bar{x}}\frac{\partial}{\partial x}\left(\frac{\partial x}{\partial \bar{x}}\frac{\partial e}{\partial x}\right)$$

and

$$\frac{\partial x}{\partial \bar{x}} = \frac{1}{1+A\cos(\omega t+\phi)},$$

one can reduce the forced Eq. (2) to

$$\frac{\partial e}{\partial t} = \frac{1}{[1+A\cos(\omega t+\phi)]^2} e_{xx} + e_{yy} - ke(e-a)(e-1) - eg,$$

$$\frac{\partial g}{\partial t} = \varepsilon(e,g)(-g - ke(e-a-1)), \tag{3}$$

which is identical with Ref. [24]: the stretching of the medium can be modeled by changing the size of the grid in the x-direction in the numerical simulation of Eq. (1).

Comparing Eqs. (1) and (3), one can see that the diffusion constant D_x [equal to 1 in Eq. (1)] has been changed to $[1+A\cos(\omega t+\phi)]^{-2}$ from the points of mathematics. Recently, there has been considerable interest in pattern formation on anisotropic reaction-diffusion systems [29], such as phase turbulence in the anisotropic complex Ginzburg-Landau equation [30], reaction-diffusion waves with sharp corners [31–33], traveling wave fragments in anisotropic excitable media [34], pattern formation processes in cardiac tissue [35]. In reaction-diffusion systems, anisotropy usually enters via the diffusion constants. In two dimensions, one can distinguish "simple" and "complex" anisotropy [29]. Simple anisotropy can usually be removed by simple scale transformations to dimensionless units. A complex anisotropy, in contrast, cannot be removed by a simple scale transformation and new pattern formation phenomena may appear. From the point of mathematics, the mechanical deformation leads to a *time-dependent* simple anisotropy in diffusion constant $D_x=[1+A\cos(\omega t+\phi)]^{-2}$, which has not been studied in detail [29]. It will be shown that the time-dependent simple anisotropy will induce appearance of new pattern formation phenomena—spiral breakup.

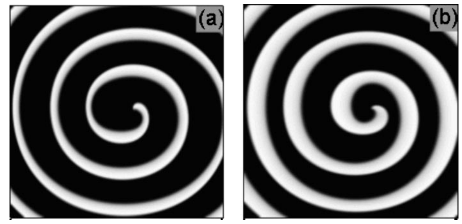

FIG. 1. Two typical states of spiral of Eq. (3) in the periodic mechanical deformation with $A=0.3$, $\omega=0.7\omega_0$, and $\phi=\pi/2$. (a) Contracted spiral. (b) Extended spiral. The spatial patterns are a gray scale plot of e.

B. Numerical results

We study the effects of mechanical deformation on spiral waves by numerical simulation of Eq. (3) for a large range for the parameters A and ω. In this paper, we will not discuss the drift of spiral waves induced by mechanical deformation of excitable media (see Ref. [25] for the resonant drift of spirals), but concentrate on the breakup of spiral waves induced by mechanical deformation. The values of the parameters used in this investigation in Eq. (3) are $k=8$, $a=0.15$, $\epsilon=0.002$, $\mu_1=0.2$, $\mu_2=0.3$. We shall refer to the space and time units of this equation as s.u. and t.u., respectively. We solve this system with explicit Euler scheme with time step $\Delta t=0.1$ t.u. and space step $\Delta x=0.5$ s.u. with no-flux boundary conditions.

For the same initial condition, we study the dynamics of spiral waves for different A and ω. Spiral waves will be periodically extended or contracted along x-direction after we switch on a relative strong mechanical deformation (Fig. 1). Generally, spiral will not break up at small deformation. We find that spiral breakup occurred spontaneously when the amplitude $A\geqslant 0.29$ and the frequency is around $0.5\omega_0$ [$\omega_0=0.3762$ is the angular frequency of the spiral of the FitzHugh-Nagumo model (1)]. Numerical results show that when we further increase the amplitude A, the range of ω for spiral breakup will be extended. Figure 2 shows different stages of the transition from spiral waves to spiral turbulence with $\omega=0.5\omega_0$, $A=0.3$. We initiate a well developed spiral wave [Fig. 2(a)] in a quiescent medium and then switch on the mechanical deformation. After the system begins the mechanical deformation, the spiral becomes contracted or extended in the direction of mechanical deformation periodically [Fig. 2(b)]. After the spiral rotates several times, it starts to break up in the regions not far from the spiral tip [Fig. 2(c)]. The region of chaotic behavior continuously extends to other regions [Figs. 2(d) and 2(e)] and eventually the system is full of spiral defects [Fig. 2(f)].

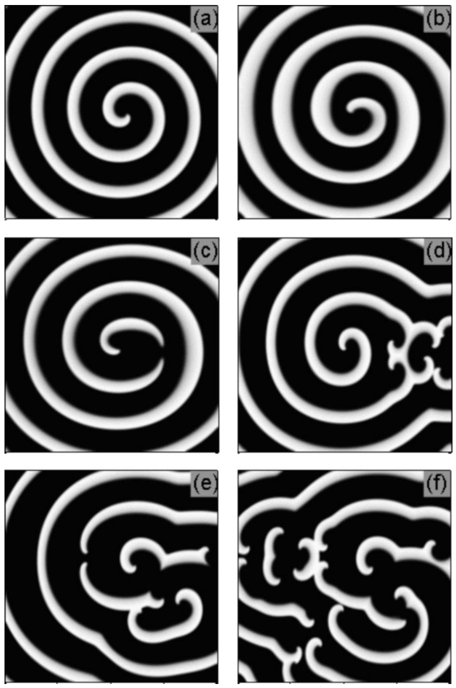

FIG. 2. Time sequence illustrating the dynamics of spiral breakup in a square lattice of side $L=35$ with $A=0.3$ and $\omega=0.5\omega_0$. (a) $t=0$, (b) $t=87$ t.u., (c) $t=99$ t.u., (d) $t=333$ t.u., (e) $t=792$ t.u., (f) $t=999$ t.u. Spirals begin to break up after some transient rotations. The resulting free ends created new spirals, which in turn break after some time, eventually giving rise to a noncoherent (turbulent) state.

The process by which the spiral breaks up in our case is shown in detail in Fig. 3. One wavefront of the spiral waves becomes unstable, and finally the wave breaks up. And after the broken waves meet together, an excited spot is formed. This spot interacts with the following wave and creates two wave breaks, which develop into two spirals.

C. The mechanism

Previous theoretical studies of wave propagation based on partial differential equations of excitable reaction-diffusion systems show that the propagation velocity of excitable wave depends on their local curvature and the curvature dependence of propagation velocity is nonlinear [26]:

$$C = C(K).$$

And there is a critical curvature K_c, such that no waves with higher curvature can propagate in a given medium: the wave front that locally exceeds the critical curvature K_c will break up in this region and two free tips will appear. In Ref. [27], the authors determined the critical curvature K_c analytically and show that the diffusion constant D dependence of K_c is

$$K_c \sim D^{-1}.$$

Therefore, the critical curvature K_c will be modulated when periodic mechanical deformation is applied, i.e., a time-dependent diffusion coefficient

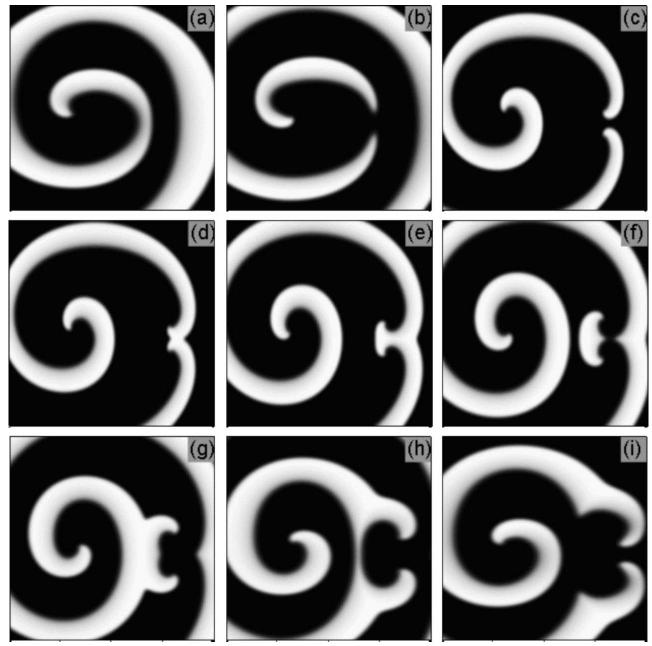

FIG. 3. Development of spiral breakup. The parameters are the same in Fig. 2.

$$D_x = [1 + A\cos(\omega t + \phi)]^{-2}; \quad D_y = 1.$$

From the simulations, we can see that the local curvature K of the wave is also changed when the mechanical deformation is applied (see Figs. 1–3). Now the mechanism of spiral breakup due to mechanical deformation can be understood as the following discussions: When the periodic mechanical deformation is applied, the local curvature K of spiral waves as well as the critical curvature K_c will change with the time periodically. When the amplitude of mechanical deformation is above a critical value and the frequency of mechanical deformation is chosen suitably, in some region the local curvature K of the spiral waves will exceed critical curvature K_c and the breakup of spiral waves will thus happen.

In Fig. 4, we give a section of the phase diagram in the

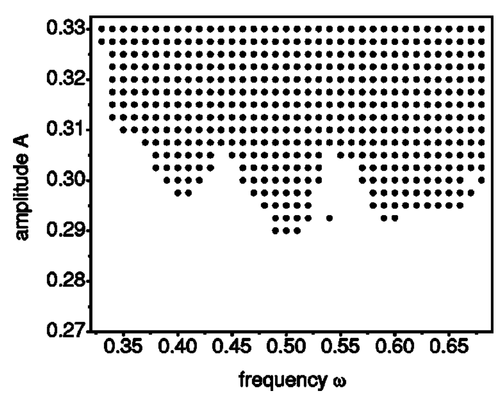

FIG. 4. Phase diagram revealing regions of deformed spiral and spiral turbulence in the $A-\omega$ plane ($A \in [0.27, 0.33]$, $\omega \in [0.33, 0.68]$) of the forced FitzHugh-Nagumo model (3). Spiral will break up at the dots. The steps of A and ω are 0.0025 and 0.01, respectively.

$A-\omega$ plane ($A\in[0.27,0.33]$, $\omega\in[0.33,0.68]$) of the force FitzHugh-Nagumo model. In the numerical simulations, the steps of A and ω are 0.0025 and 0.01, respectively. One can see that spiral begin to break up at $\omega=0.5\omega_0$ when we increase the amplitude gradually. The range of ω for spiral breakup becomes wider when A is increased.

III. SPIRAL BREAKUP IN THE OREGONATOR MODEL

To check whether spiral breakup induced by mechanical deformation is sensitively model dependent, we also study the influence of mechanical deformation on spiral waves in the Oregonator model [36]:

$$\frac{\partial u}{\partial t}=\frac{1}{\epsilon}\left[u-u^2-fv\frac{u-q}{u+q}\right]+\nabla^2 u,$$

$$\frac{\partial v}{\partial t}=u-v. \quad (4)$$

Here variables u and v represent the concentrations of the autocatalytic species $HBrO_2$ and the catalyst in the Belousov-Zhabotinsky (BZ) reaction. With medium mechanical deformation, the Oregonator model will be modified as

$$\frac{\partial u}{\partial t}=\frac{1}{\epsilon}\left[u-u^2-fv\frac{u-q}{u+q}\right]+\frac{1}{[1+A\cos(\omega t+\phi)]^2}u_{xx}+u_{yy},$$

$$\frac{\partial v}{\partial t}=u-v. \quad (5)$$

Numerical simulations of above equations show that spiral will break up when the amplitude of mechanical deformation is larger than 0.37. Being the same as the results of the FitzHugh-Nagumo model, spiral begin to break up almost at $\omega=0.5\Omega_0$ [$\Omega_0=3.1416$ is the angular frequency of the spiral of the Oregonator model (4)] when we increase the force gradually. Figure 5 gives an example of spiral breakup in the forced Oregonator model (5).

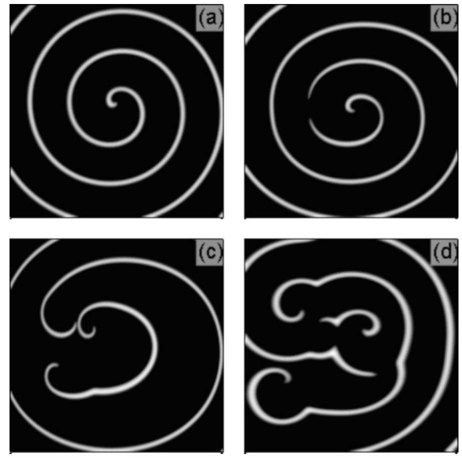

FIG. 5. The sequence of pictures shows the evolution of a spatial pattern in the numerical simulation of the forced Oregonator model (5) with $\epsilon=0.05$, $f=2.0$, $q=0.002$, and $A=0.38$, $\omega=0.52\Omega_0$, and $\phi=\pi/2$. The spatial patterns are a gray scale plot of u.

IV. CONCLUSIONS

We have studied spiral breakup under the influence of the mechanical deformation of the medium. It is found that spiral is not stable and begin to break up when the mechanical deformation is strong. Although the mechanical deformation is one kind of time-dependent simple anisotropies [29], it can induce appearance of new pattern formation phenomena: spiral breakup. The mechanism of spiral breakup induced by mechanical deformation is intuitively understood: the local curvature K of the wave and the critical curvature K_c are both modulated when the periodic mechanical deformation is applied, and the wave is broken when the local curvature K exceeds the critical curvature K_c. Since the heart is contracting all the time, the mechanical deformation maybe is one reason for spiral breakup in cardiac muscle, which is responsible for life-threatening situations.

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